Electronic and Signal Processing Exam

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Information

Welcome to the Electronics and Signal Processing exam. Please read carefully the information below.

How to write your solution
Please use a **pen** and not a pencil. Make sure your **hand writing** is understandable by others. **Drawings** do not need to be beautiful/perfect but it is important that they are easy to understand and there are no ambiguities (e.g. a gate which could be an OR or an AND, but it is not clear which one it is, label it to avoid confusion).
Each **solution** has to be justified and **the steps to get there have to be explicitly written down**, only providing the final outcome will lead to zero points. On **every page** please indicate which problem you are working on. If you separate the pages please indicate your name and student ID on **every page**.
For your convenience, you can find a page with basic equations related to the course material at the end of this document.

Full name: .... Student ID: ....

Problem 1 (17 points)

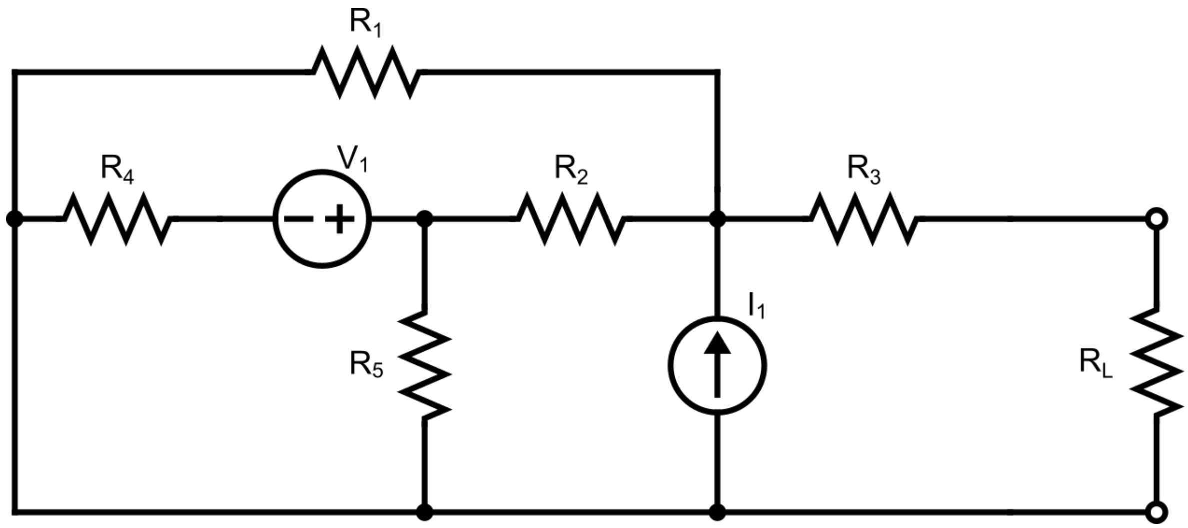

Figure 1: Resistor network.

Consider the circuit in Figure 1 and the related **parameters**: $R_1 = 2\ k\Omega$, $R_2 = 2\ k\Omega$, $R_3 = 2\ k\Omega$, $R_4 = 6\ k\Omega$, $R_5 = 3\ k\Omega$, $V_1 = 7.5\ V$, $I_1 = 2.5\ mA$.

(a) (4 points) Using the parameters provided above, calculate the equivalent resistance R_{eq} seen by R_L (consider R_L an open circuit and calculate the resistance between the two open terminals).

(b) (10 points) Using the parameters provided above, calculate the Norton short circuit current I_{no} seen by R_L.

(c) (1 points) Draw the Norton equivalent circuit including R_L as load.

(d) (2 points) Given that $R_L = 10k\Omega$, calculate the current flowing through and the voltage across the load resistance R_L. If you do not have values for I_{no} and R_{eq} calculated from the previous questions use the following **incorrect** values: $I_{no} = 1\ mA$ and $R_{eq} = 2\ k\Omega$.

Problem 1 - Solution

Point a)

If we replace the generators with their internal resistors, (that means the voltage source acts as a short circuit and the current source acts as an open circuit, see Fig. 2) we see that the equivalent resistance seen by R_L is:

$$\begin{aligned}
R_{eq} &= R_3 + R_1//(R_2 + R_4//R_5) \\
&= R_3 + R1//\left(R_2 + \frac{R_4R_5}{R_4+R_5}\right) \\
&= R_3 + \frac{R_1R_2 + \frac{R_1R_4R_5}{R_4+R_5}}{R_1 + R_2 + \frac{R_4R_5}{R_4+R_5}} \\
&= R_3 + \frac{R_1(R_2R_4 + R_2R_5 + R_4R_5)}{(R_1+R_2)(R_4+R_5) + R_4R_5} \\
&= 2\text{k}\Omega + \frac{2(12+6+18)}{4\cdot 9 + 18}\ \text{k}\Omega = (2 + \frac{4}{3})\text{k}\Omega
\end{aligned}$$

So $R_{eq} = \underline{\underline{\frac{10}{3}\text{k}\Omega}}$.

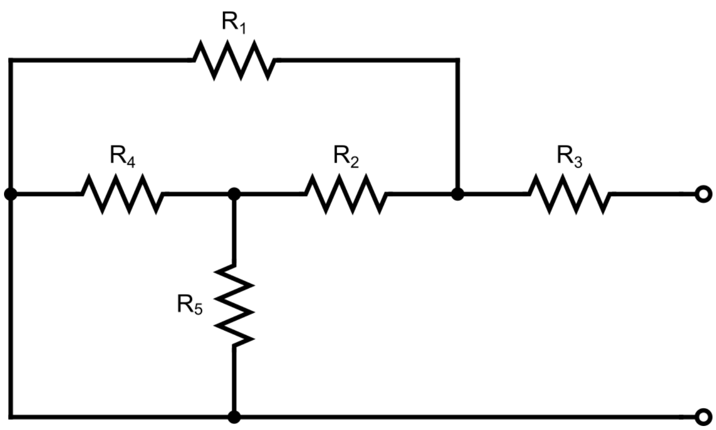

Figure 2: Circuit from question 1 redrawn with the sources removed to find the equivalent resistance

Point b)

Solution I

The equivalent generator can be calculated using the superposition principle. Replacing the current source with an open circuit, we can redraw as in Fig. 3. In order to now find I_{SCV}, the contribution to I_{SC} from the voltage source, we first have to find the total current from the voltage source I_{tot}. To help with this, we define $R_a = R_2 + R_3//R_1 = 2\text{k}\Omega + 2//2\text{k}\Omega = 3\text{k}\Omega$. This gives us

$$I_{tot} = \frac{V_1}{R_4 + R_5//R_a} = \frac{7.5\text{V}}{(6+3//3)\text{k}\Omega} = 1\text{mA}$$

We now recognize $R_5//R_a$ as an equal current divider, as well as $R_3//R_1$, such that $I_{SCV} = I_{tot}/4 = \underline{\frac{1}{4}\text{mA}}$.

Alternatively, the current divider formula can be used to find that

$$I_{SCV} = \frac{R_5}{R_a + R_5}\frac{R_1}{R_1 + R_3}\frac{V_1}{R_4 + R_5//R_a} = \frac{R_5R_1}{(R_3+R_1)(R_5R_a + R_4R_a + R_4R_5)}V_1$$

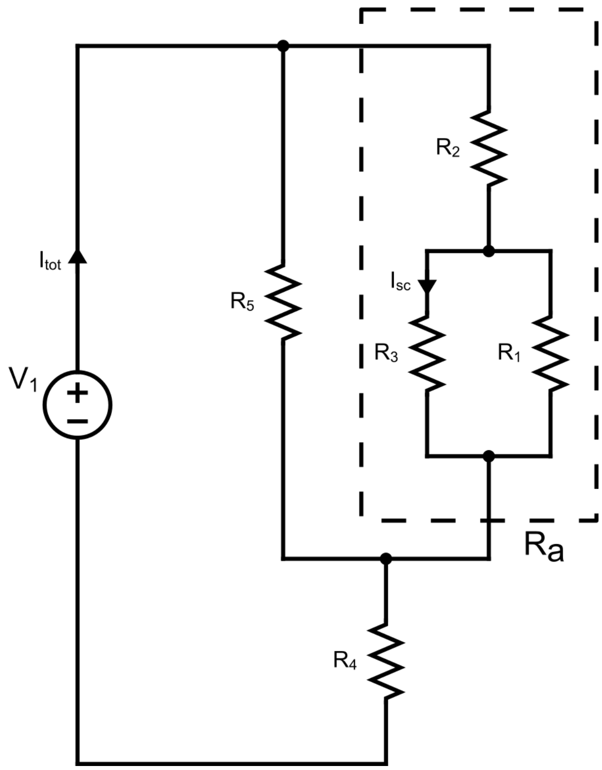

Figure 3: Norton equivalent current: part 1

which solves to the same thing.

To find the contribution from the current source, we replace the voltage source with a short-circuit, as can be seen in Fig. 4. To find I_{SCI}, we recognize that the current from I_1 gets divided between the left and right part of the circuit, so we define a resistance R_b to be the resistance of the left side of the circuit, which is the same as $R_b = R_{eq} - R_3 = \frac{4}{3}\text{k}\Omega$. We then use the current divider formula to find

$$I_{SCI} = I_1 \frac{R_b}{R_3 + R_b} = 7.5 \frac{\frac{4}{3}}{\frac{10}{3}} \text{mA} = 0.4 \cdot 2.5\text{mA} = \underline{1\text{mA}}$$

This puts our total $I_{SC} = \underline{\underline{\frac{5}{4}\text{mA}}}$

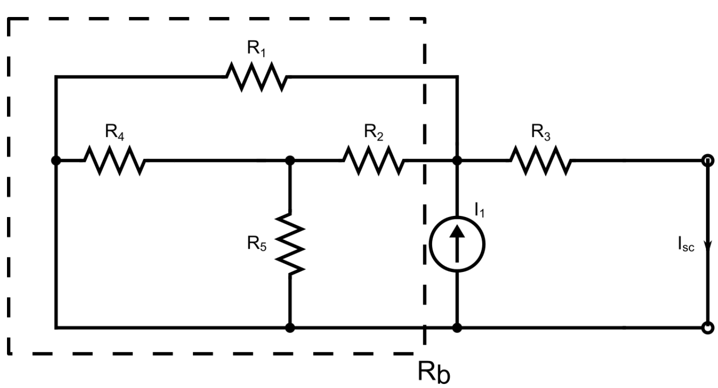

Figure 4: Norton equivalent current: part 2

Solution II

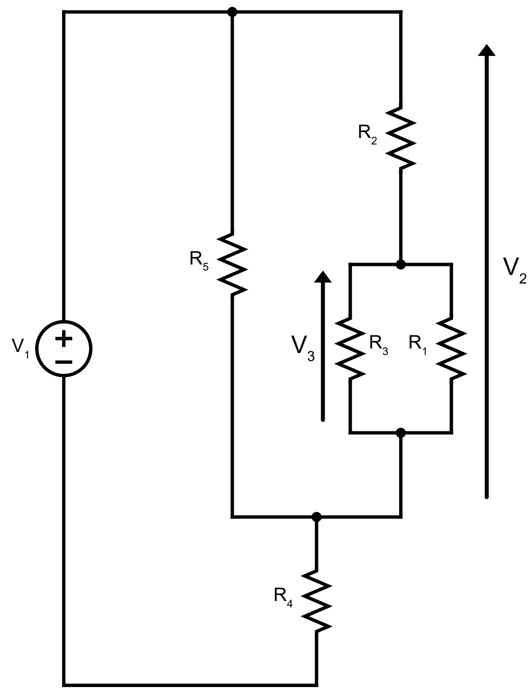

Figure 5: Norton equivalent current: method II, part 1

Alternatively, we can use voltage dividers to find I_{SCV}. Starting with the voltage source, we replace the current source with an open circuit, and redraw the circuit like in Figure 5. We now use the voltage divider to find

$$V_2 = \frac{R_5//(R_1 + R_2//R_3)}{R_5//(R_1 + R_2//R_3) + R_4} V_1 = \frac{3k//3k}{3k//3k + 6k} V_1 = \frac{3}{15} V_1$$

We can also find that

$$V_3 = \frac{R_2//R_3}{R_2//R_3 + R_1} V_2 = \frac{1k}{1k + 2k} V_2 = \frac{1}{3} V_2$$

Then finally to find I_{SCV}:

$$I_{SCV} = \frac{V_3}{R_3} = \frac{1}{2k}\frac{1}{3}\frac{3}{15} V_1 = \frac{1}{30k} 7.5\text{V} = \underline{\frac{1}{4}\text{mA}}$$

For the current source, we can redraw the circuit like in Figure 6, and find the total resistance to be:

$$R_{\text{tot}} = R_3//R_1//(R_2 + R_5//R_4) = 2k//2k//(2k + 3k//6k) = 1k//(2k + 2k) = 1k//4k = \frac{4}{5}\text{k}\Omega$$

Then from there we calculate the total voltage: $V_{R_3} = \frac{5}{2} \cdot \frac{4}{5} = 2\text{V}$, from which we find

$$I_{SCI} = \frac{V_{R_3}}{R_3} = \frac{2}{2} = \underline{1\text{mA}}$$

This puts our total $I_{SC} = \underline{\underline{\frac{5}{4}\text{mA}}}$

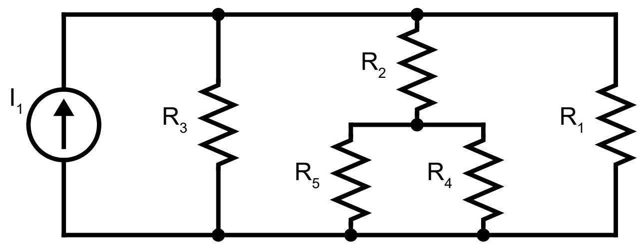

Figure 6: Norton equivalent current: method II, part 2

Solution III

It is also possible to use nodal analysis to find I_{SC} directly. For this, we define voltages V_2 and V_3, as shown in Fig. 7. By using the fact that the voltage over R_4 is $V_2 - V_1$, we can now set up the equations:

$$\mathbf{V_2} : \frac{V_2}{R_5} + \frac{V_2 - V_1}{R_4} + \frac{V_2 - V_3}{R_2} = 0$$
$$\mathbf{V_3} : \frac{V_3 - V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_3}{R_1} = I_1$$

Filling in the numbers and refactoring gives:

$$\mathbf{V_2} : \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right) V_2 - \frac{1}{2} V_3 = \frac{1}{6} \cdot 7.5\text{V}$$
$$\mathbf{V_3} : -\frac{1}{2} V_2 + \frac{3}{2} V_3 = 2.5\text{mA} \cdot 1\text{k}\Omega = 2.5\text{V}$$

or

$$\mathbf{V_2} : 6V_2 - 3V_3 = \cdot 7.5\text{V}$$
$$\mathbf{V_3} : -V_2 + 3V_3 = 5\text{V}$$

which solves for V_3 as

$$15V_3 = 7.5 + 6 \cdot 5\text{V} \Rightarrow V_3 = \frac{5}{2}\text{V}$$

so that we get

$$I_{SC} = \frac{V_3}{R_3} = \underline{\underline{\frac{5}{4}\text{mA}}}$$

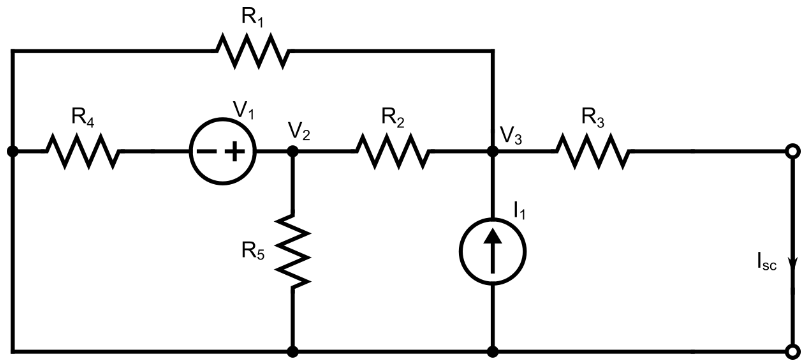

Figure 7: Norton equivalent current: method III, nodal analysis

Point c)

We can use the answers from points a) and b) to redraw the circuit as shown in Fig. 8. The sources and resistors (except for R_L) have been replaced by a single current source in parallel with a single resistor such that the voltage between the nodes around R_L is equivalent to the original situation.

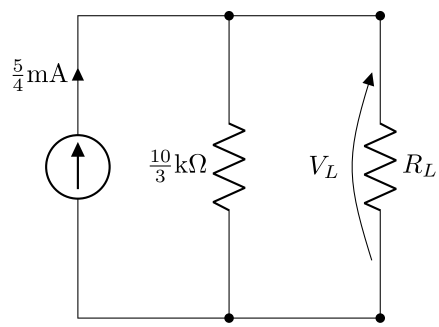

Figure 8: Norton equivalent circuit

Point d)

We once again use a current divider to find

$$I_L = I_{SC}\frac{R_{eq}}{R_{eq}+R_L} = \frac{5}{4}\frac{\frac{10}{3}}{10+\frac{10}{3}} = \frac{5}{4}\frac{10}{10+30} = \frac{5}{16}\text{mA}$$

Alternatively:

$$V_L = I_{SC}\ R_L//R_{eq} = I_{SC}\frac{R_L R_{eq}}{R_L+R_{eq}}$$

$$I_L = \frac{V_L}{R_L}$$

For provided values:

$$V_L = 1\cdot\frac{10\cdot 2}{10+2}\text{V} = \frac{20}{12}\text{V} = \frac{5}{3}\text{V}$$

$$I_L = \frac{V_L}{R_L} = \frac{5}{3}\frac{1}{10}\text{mA} = \frac{1}{6}\text{mA}$$

Remarks

This question is comparable to the Top Problem from Week 2. The number and type of sources and resistors is identical (1 voltage source and 1 current source, 4 resistors).

Problem 2 (20 points)

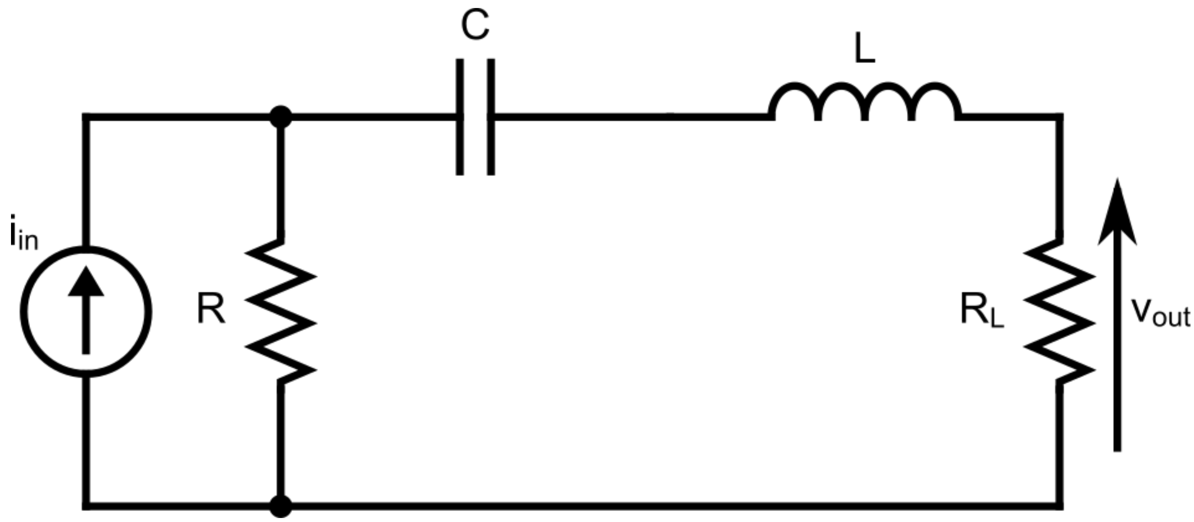

Figure 9: RLC circuit.

Consider the circuit in Figure 9. Assume **sinusoidal** regime, **ideal** components and the following **parameters**: $C = 25\ mF$, $L = 0.5\ H$, $R = 1\ \Omega$, $R_L = 5\ \Omega$, $\omega_0 = 20$ rad s^{-1}.

(a) (3 points) **Without** any calculation, but **only** reasoning about the behavior of each element in the circuit (to be mentioned explicitely), describe the behavior of

$$H(\omega) = \frac{v_{\text{out}}}{i_{\text{in}}}$$

for low ($\omega \to 0$) and high ($\omega \to \infty$) frequencies.

(b) (11 points) Derive the transfer function $H(\omega)$. Using the parameters provided above, calculate its value in ω_0: $H(\omega_0)$.

(c) (4 points) At what point is the amplitude of the output signal, V_{out}, maximum? Derive and calculate the value of ω in rad s^{-1}.

(d) (2 points) Taking R_L to be a removable load, find the equivalent impedance of the supply circuit.

Problem 2 - Solution

Point a)

For low frequencies ($\omega \to 0$ rad/s) the capacitor impedance $Z_C \to \infty$, while the inductor impedance $Z_L \to 0$.
For high frequencies ($\omega \to \infty$ rad/s) the capacitor impedance $Z_C \to 0$, while the inductor impedance $Z_L \to \infty$.
In both cases, the impedance of the three components in series $Z_a = Z_C + Z_L + Z_{R_L} \to \infty$. Therefore no current goes through the right branch, therefore $v_{\text{out}} = i_{\text{out}} R_L \to 0$, therefore $\underline{\underline{H(\omega) \to 0\,\mathrm{V\,A^{-1}}}}$.

Point b)

Method I

Recognise that we have a current divider here. Denote the impedance of the three parallel components as

$$Z_a = Z_C + Z_L + Z_{R_L}$$

By the current divider equation, the current through the right branch is then

$$i_{\text{out}} = i_{\text{in}} \frac{R}{R + Z_a}$$

and so the transfer function is given by

$$\begin{aligned}
H(\omega) &= v_{\text{out}}/i_{\text{in}} \\
&= R_L i_{\text{out}}/i_{\text{in}} \\
&= R_L \frac{R}{R + Z_a} \\
&= \frac{R R_L}{R + Z_C + Z_L + Z_{R_L}} \\
&= \frac{R R_L}{R + R_L + (j\omega C)^{-1} + j\omega L} \\
&= \frac{R R_L}{R + R_L + j[\omega L - \frac{1}{\omega C}]}
\end{aligned}$$

We can now substitute in the various parameters, giving

$$H(\omega) = \frac{5}{6 + j[0.5\omega - \frac{40}{\omega}]}$$

Finally, for ω_0, we get

$$H(\omega_0) = \frac{5}{6 + j[10 - \frac{40}{20}]} = \frac{5}{6 + 8j} = \underline{\underline{0.3 - 0.4j\ \mathrm{V\,A^{-1}}}}$$

and correct units are required to be awarded full marks.

Method II

Alternatively, we can find the voltage over the current source, v_{in} to be (where Z_a is the same as described above):

$$v_{\text{in}} = i_{\text{in}}(R//Z_a) = i_{\text{in}} \frac{R Z_a}{R + Z_a}$$

We then recognize that R_L forms a voltage divider together with Z_C and Z_L to get:

$$v_{\text{out}} = v_{\text{in}} \frac{R_L}{Z_a} = i_{\text{in}} \frac{R Z_a}{R + Z_a} \frac{R_L}{Z_a} = i_{\text{in}} \frac{R R_L}{R + Z_a}$$

from which we can continue as described above in Method I.

Point c)

To maximise $|H(\omega)|$ w.r.t ω, we use our previous expression for $H(\omega)$, recognising that it is maximised when the imaginary part of the denominator equals 0, so when:

$$\begin{aligned}
\omega L &= \frac{1}{\omega C} \\
\omega^2 &= \frac{1}{LC} \\
\omega &= \frac{1}{\sqrt{LC}} \\
&= \underline{\underline{4\sqrt{5}\,\mathrm{rad}\,s^{-1}}} \approx \underline{\underline{8.94\,\mathrm{rad}\,s^{-1}}}
\end{aligned}$$

and of course units are necessary for full marks.

Point d)

As with Northon/Thévenin equivalent circuits, we set i_{in} to be open circuit. Then from R_L's perspective, the circuit is just the three remaining components in series:

$$R_{eq}(\omega) = R + Z_C + Z_L = R + j\big(\omega L - \frac{1}{\omega C}\big) = 1 + j(0.5\omega + \frac{40}{\omega})$$

or, if taken at ω_0 (due to ambiguity in the question, both were counted as correct):

$$R_{eq}(\omega_0) = 1 + 8j\,\Omega$$

Remarks

This question is in complexity and type of tasks comparable to top problem from week 3. Finding limits by inspection and only reasoning as in subquestion a) was extensively practiced during the lectures. For more info see *Electronics: a Systems Approach* chapter 6.6 or *Electronics for Physicists* chapter 3.

Problem 3 (15 points)

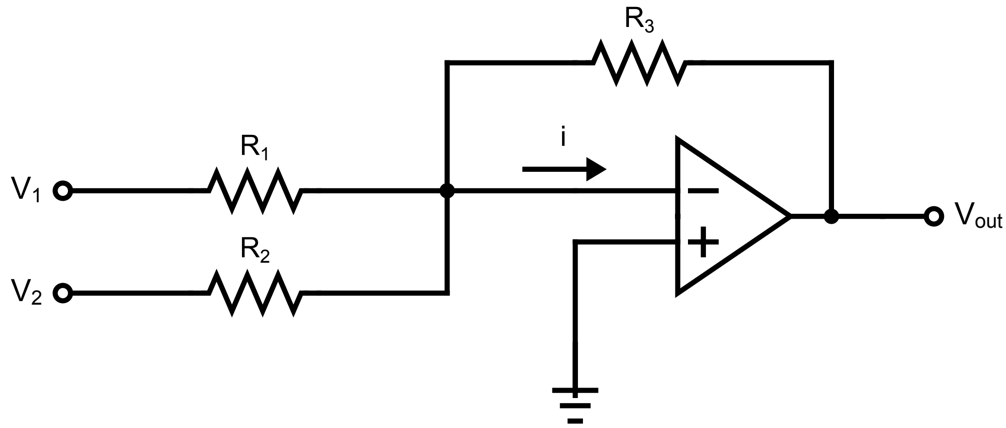

Figure 10: Circuit with an **ideal** op-amp.

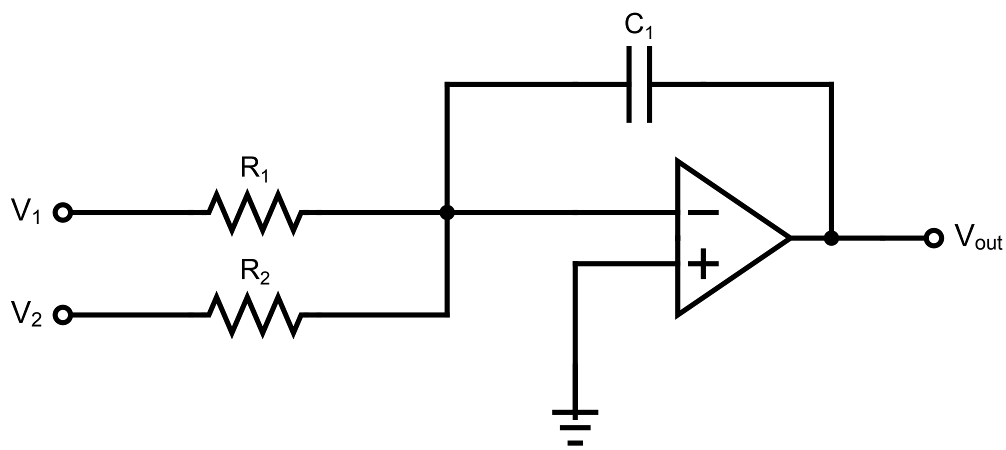

Figure 11: Circuit with an **ideal** op-amp and capacitor.

Consider the circuit in Figure 10. Keep your solutions in terms of the variables defined unless stated otherwise.

(a) (1 points) What is the current i? Give a reason for your answer.

(b) (6 points) V_1 and V_2 are constant DC voltages. Find an expression for the output voltage V_{out}.

Now consider the circuit in Figure 11. Again, keep your solutions in terms of the variables defined unless stated otherwise.

(c) (8 points) Find an expression for V_{out} given that V_1 and V_2 are AC inputs. What is V_{out} for $\omega \to \infty$? How can the same conclusion be reached without having an expression for V_{out}?

Problem 3 - Solution

Part a)

$\underline{\underline{i = 0}}$. The input current in an ideal op-amp is so small that it is assumed to be zero.

Part b)

Method I

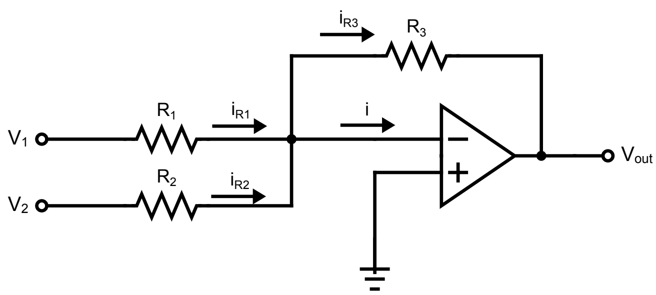

Figure 12: Current definitions for KCL for solving the op-amp circuit

Since the op-amp is ideal, we can use fact that $V_- = V_+ = 0\text{V}$. Now, since $i = 0$, we can use Kirchoff's current law (KCL) to find (see Fig. 12 for the current definitions used):

$$\begin{aligned} i_{R_3} &= i_{R_1} + i_{R_2} \\ \frac{0 - V_{\text{out}}}{R_3} &= \frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} \\ V_{\text{out}} &= \underline{\underline{-R_3(\frac{V_1}{R_1} + \frac{V_2}{R_2})}} \end{aligned}$$

From which we see that this op-amp is a summing amplifier.

Alternatively, you could start with superposition, recognizing that the total V_{out} is the sum of the contributions from V_1 and V_2. To find the contribution from V_1, we set $V_2 = 0$, meaning that the current $i_{R_2} = 0$ as $V_- = V_+ = 0$ too. Then because $i_{R_2} = 0$, we can use a voltage divider[1]:

$$\begin{aligned} V_- = 0 &= V_1 + (V_{\text{out}} - V_1)\frac{R_1}{R_1 + R_3} \\ -V_1 &= (V_{\text{out}} - V_1)\frac{R_1}{R_1 + R_3} \\ V_{\text{out}} &= -\frac{R_1 + R_3}{R_1}V_1 + V_1 = -\frac{R_3}{R_1}V_1 \end{aligned}$$

Similarly when we set $V_1 = 0$, the output current would be

$$V_{\text{out}} = -\frac{R_3}{R_2}V_2$$

Summing these two solutions, the output voltage is found to be

$$V_{\text{out}} = \underline{\underline{-R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)}}$$

[1] it's also possible to just use KCL here, but for the purpose of demonstration we use the voltage divider

Part c)

By writing C_1 as a complex impedance $Z_{C_1} = \frac{1}{j\omega C_1}$, we can solve this question the same as we did for part b). First we recognize again that $V_+ = V_- = 0$, and that the current into the $-$ terminal of the op-amp is 0A. We then use KCL to find:

$$\begin{aligned} i_{C_1} &= i_{R_1} + i_{R_2} \\ \frac{0 - V_{\text{out}}}{Z_{C_1}} &= \frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} \\ V_{\text{out}} &= -Z_{C_1}(\frac{V_1}{R_1} + \frac{V_2}{R_2}) \\ V_{\text{out}} &= \underline{\underline{\frac{j}{\omega C_1}(\frac{V_1}{R_1} + \frac{V_2}{R_2})}} \end{aligned}$$

As we now take the limit $\omega \to \infty$ we can see that $V_{\text{out}} \to 0$ This can also be seen by realising that the capacitor becomes essentially a short-circuit in the limit $\omega \to \infty$. This connects V_{out} to V_-, which is a virtual ground, equal to $V_+ = 0$.

Remarks

The use of KCL and complex impedances for op-amp circuits was used in problem 10.7 from *Electronics for Physicists*, from the tutorial in week 5. Summing amplifiers are discussed in chapter 16.4.4 (page 287) of *Electronics a Signals Approach*, and Chapter 10, example 5, page 204 of *Electronics for Physicists*, and the general concept of superposition principle for op-amps was discussed in the top problem of week 4.

Problem 4 (18 Points)

Legend: $A \cdot B$ = AND, $A + B$ = OR, $\overline{A}$ = NOT A

(a) (9 points) Write the Boolean expression implemented by the circuit in Fig. 13 and fill in the Karnaugh map on the right side of the figure.

Using the Karnaugh map, derive an optimised **logic** expression to implement the above circuit function. No need to draw the resulting circuit.

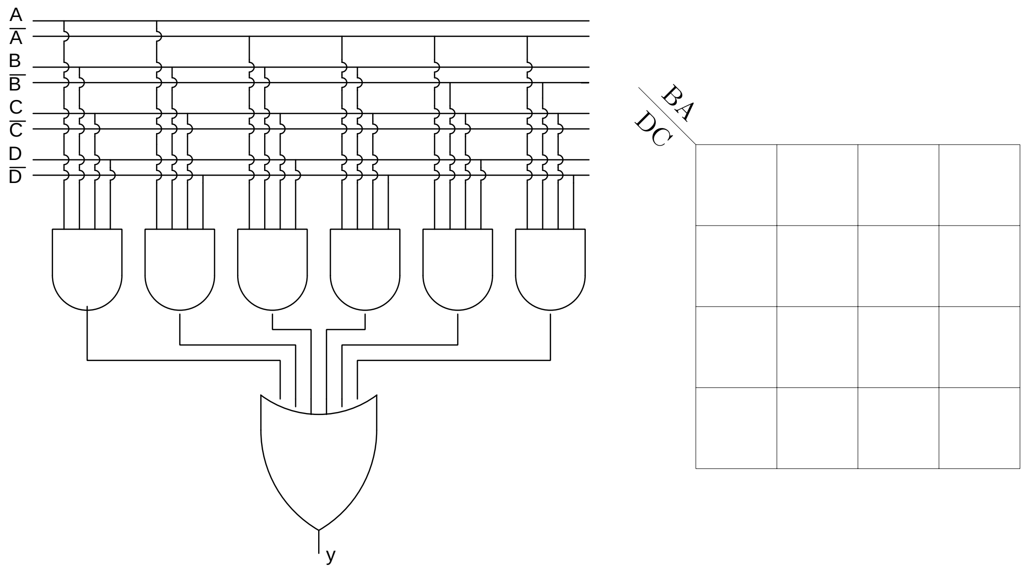

Figure 13: Logic circuit and Karnaugh map to be filled out.

(b) (9 points) Convert the following Boolean expression

$$Y = (A \cdot B \cdot (C + D)) + (\overline{A} \cdot (B + C))$$

to NOR logic using Boolean algebra. The final expression must require **only 2-input NOR gates** (NOR2), but there is no constrain on the number of gates needed. Draw the circuit to implement the combinational logic corresponding to the derived Boolean expression.

Problem 4 - Solution

Point a)

The Karnaugh map is drawn with gray code, so only one digit changes per step, with 2 variables vertical and 2 horizontal. Each of the given terms (e. g. $\overline{D} \cdot \overline{C} \cdot A \cdot \overline{B}$) appears as a one in the map (e.g. in the cell with $B = C = D = 0$ and $A = 1$). All other positions are filled with a zero. Mark all groups of 16 (all variables reduced), 8 (3 variables reduced), 4 (two variables reduced), 2 (one variable reduced) that you can find, individual positions can be used in multiple groups. Remove any redundant group. Groups can be formed over the edge.

$$y = (A \cdot B \cdot C \cdot D) + (A \cdot B \cdot C \cdot \overline{D}) + (\overline{A} \cdot B \cdot C \cdot D) + (\overline{A} \cdot B \cdot C \cdot \overline{D}) + (\overline{A} \cdot \overline{B} \cdot C \cdot D) + (\overline{A} \cdot \overline{B} \cdot C \cdot \overline{D})$$

DC \ BA	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	1	0	1	1
10	0	0	0	0

$$y = (green) + (red)$$
$$y = (B \cdot C) + (\overline{A} \cdot C)$$

$$A \cdot B = \text{AND}$$
$$A + B = \text{OR}$$
$$\overline{A} = \text{NOT A}$$

Remarks

The Karnaugh map for this question is comparable to the one provided in the solution of the top problem of Week 8, as it is the task to find the reduced formula.

Point b)

$$Y = (A \cdot B \cdot (C + D)) + (\overline{A} \cdot (B + C))$$

The De Morgan's laws can be used to turn all gates into 2-input NOR gates. First change the formula to only have 2-input gates

$$y = ((A \cdot B) \cdot (C + D)) + (\overline{A} \cdot (B + C))$$

Introduce double inversion to the inner most 2-input AND expressions:

$$y = (\overline{\overline{(A \cdot B)}} \cdot (C + D)) + (\overline{A} \cdot (B + C))$$

Swap gates from 2-input AND gates to 2-input NOR gates by inverting the inputs:

$$y = (\overline{(\overline{A} + \overline{B})} \cdot (C + D)) + (\overline{A} \cdot (B + C))$$

Introduce double inversion to the remaining 2-input AND expressions:

$$y = \overline{\overline{((\overline{\overline{A} + \overline{B}}) \cdot (C + D))}} + \overline{\overline{(\overline{A} \cdot (B + C))}}$$

Swap gates from 2-input AND gates to 2-input NOR gates by inverting the inputs:

$$y = \overline{((\overline{\overline{\overline{A} + \overline{B}}}) + \overline{(C + D)})} + \overline{(\overline{\overline{\overline{A}}} + \overline{(B + C)})}$$

Remove obsolete double inversion on single variables:

$$y = \overline{((\overline{\overline{\overline{A} + \overline{B}}}) + \overline{(C + D)})} + \overline{(A + \overline{(B + C)})}$$

Add double inversion to the outer 2-input OR gate:

$$y = \overline{\overline{\overline{((\overline{\overline{\overline{A} + \overline{B}}}) + \overline{(C + D)})} + \overline{(A + \overline{(B + C)})}}}$$

Draw the circuit, use a NOR2 with both inputs connected together to implement a NOT gate. The resulting circuit is shown in Fig. 14.

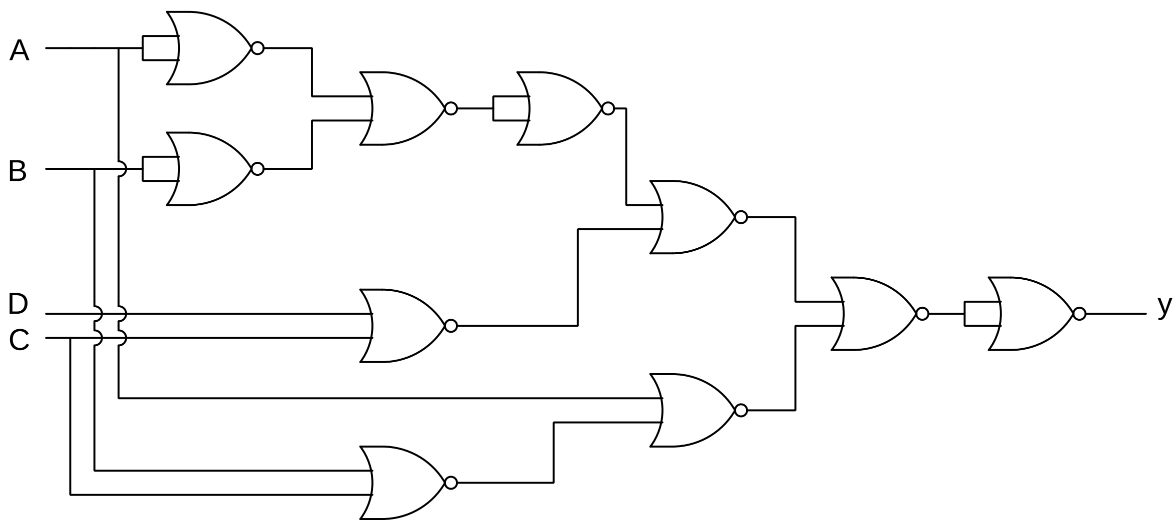

Figure 14: The 2-input NOR circuit.

Remarks

The use of the Morgan's laws for the translation of arbitrary logic functions to NAND and NOR logic was extensively covered in the lecture (both in graphical and algebraic form). Top problem from Week 7 has covered a similar problem, providing more practice with Boolean algebra operations.

Electronics and Signal Processing - Formula Sheet

Ohm's law: $V = ZI$ Capacitors: $V = \frac{Q}{C} = \frac{1}{C}\int I \, \mathrm{d}t$ Inductors: $V = L\,\frac{\mathrm{d}I}{\mathrm{d}t}$

Complex impedance: $Z_R = R$, $Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$ Reactance: $X = \mathfrak{Im}(Z)$

Quality ratio : $Q = \frac{f_0}{B}$ (for bandwidth B and resonance frequency f_0)

Root-mean-square voltage of sinusoidal signal = 0.707 of amplitude

Voltage gain: $A_v = \frac{V_o}{V_i}$ dB voltage gain $= 20\log_{10}(\frac{V_o}{V_i})$

Closed loop gain: $G = \frac{A}{1+AB}$, where A is the forward gain and B is the feedback gain.

Output voltage of an OpAmp: $V_{out} = A\,(V_+ - V_-)$

Characteristic impedance of a cable: $Z_{eq} = \sqrt{\frac{r}{2\omega c}}(1-j)$ (RC cable), $Z_{eq} = \sqrt{\frac{l}{c}}$ (LC cable)

Speed of signal in a cable: $v = \frac{1}{\sqrt{lc}}$ (LC cable)

Effective impedance seen by a source connected to a cable (length Λ, Z_0) and a load with impedance Z: $Z_{eff} = Z_0\frac{Z - jZ_0 tan(k\Lambda)}{Z_0 - jZ tan(k\Lambda)}$, where $k = \frac{2\pi}{\lambda} = \omega\sqrt{lc}$

Boolean Algebra

- Commutative laws: $AB = BA$, $A + B = B + A$
- Distributive laws: $A(B+C) = AB + AC$, $A + BC = (A+B)(A+C)$
- Associative laws: $A(BC) = (AB)C$, $A + (B+C) = (A+B) + C$
- Absorption law: $A + AB = A\,(A+B) = A$
- De Morgan's laws: $\overline{A+B} = \overline{A}\cdot\overline{B}$, $\overline{AB} = \overline{A} + \overline{B}$
- Other: $A + \overline{A}\cdot B = A + B$, $A\,(\overline{A} + B) = AB$

Complex Numbers Algebra

$|z| = \sqrt{\mathfrak{Re}(z)^2 + \mathfrak{Im}(z)^2}$

$e^{j\phi} = \cos(\phi) + j\sin(\phi) \implies e^{j\frac{\pi}{2}} = j$, $e^{-j\frac{\pi}{2}} = -j$, $e^{j\pi} = -1 = j^2$

$\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$ $\sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2j}$